

5D MULTIVERSE ARCHITECTURE: KLEIN-BOTTLE TOPOLOGICAL SLICES, AXIONIC PHASE FIBERS, AND INSTANTON ROUTER SCATTERING AT THE WHITE HOLE ORIGIN

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August 5, 2026

ABSTRACT

We present a 5-dimensional geometric model governing the large scale structure, topology, and evolution of the multiverse, extending the 13-axiom White Hole cyclic cosmology framework of Choudhary [1]. The global 5D manifold $\mathcal{M}^5$ is parameterized by four spacetime coordinates (x, y, z, t) and an independent 5th canonical quantum-phase coordinate $\Phi \in [0, 2\pi)$ along a compact $U(1)_{\text{cosmic}}$ fiber. The phase coordinate Φ is coupled to a physical **Cosmic Axion Field** $\Theta(x)$, whose effective potential $V(\Theta, \rho) \propto \langle \hat{R}(\rho) \rangle (1 - \cos \Theta)$ is derived directly from the Loop Quantum Cosmology Ricci scalar curvature $R(\rho) = 8\pi G(\rho + P/c^2)(1 - \rho/\rho_{\text{Planck}})$. Each 4D universe slice is constructed as a non-orientable twisted Klein-bottle manifold K^4 , enforcing a finite expansion ceiling R_{max} , driving cosmic recycling through a 5D White Hole origin, and guaranteeing a strictly unidirectional, forward-marching arrow of time ($t \rightarrow +\infty$) without Closed Timelike Curves (CTCs). We derive the exact 5D gravitational instanton action $S_{\text{instanton}}(\rho) = \frac{3\pi^2}{4}(1 - \rho/\rho_{\text{Planck}})|\Delta N|$, where the coefficient $\frac{3\pi^2}{4} = \frac{3}{8}\text{Vol}(S^3)$ is calculated from the $D = 5$ gravitational curvature tensor contraction. We prove that the quantum transition scattering amplitude $\mathcal{S}_{N_1 N_2} \propto \exp(-S_{\text{instanton}}/\hbar)$ vanishes during low-density expansion ($\rho \ll \rho_{\text{Planck}}$), but goes to order unity $\mathcal{S}_{N_1 N_2} \sim \mathcal{O}(1)$ at the Planck density ceiling ($\rho \rightarrow \rho_{\text{Planck}}$) as the background Ricci scalar vanishes ($R(\rho) \rightarrow 0$), collapsing the potential barrier. The 5D White Hole acts as a non-perturbative **Quantum Instanton Router**, re-sorting and redistributing topological winding sectors during each cosmic cycle.

Keywords 5D multiverse · Kaluza-Klein fiber bundle · Klein bottle topology · cosmic axion field · instanton router · White Hole bounce

1 Introduction: From 4D White Hole Bounce to 5D Fiber Geometry

In recent work, Choudhary [1] introduced a 13-axiom **White Hole Cyclic Cosmology** framework resolving Penrose's Initial Entropy Problem [2] ($P_{\text{initial}} \sim 10^{-10^{123}}$) by replacing the Big Bang singularity with a quantum-geometry bounce of a monolithic universal black hole at the Planck density ceiling ($\rho_{\text{Planck}} \approx 5.16 \times 10^{96} \text{ kg/m}^3$). While this 4D model establishes gravitational entropy zeroing ($S_{\text{grav}} \rightarrow 0$), time-reversed White Hole repulsive anti-gravity ($\frac{d^2 r}{dt^2} > 0$), and dynamical dark energy recollapse ($w(z) \neq -1$), it leaves fundamental questions regarding global higher dimensional structure, universe branch separation, and phase space re-sorting during the bounce phase.

In this paper, we extend the 4D cyclic framework into a complete 5-dimensional spacetime architecture $(\mathcal{M}^5, G_{AB})$. We introduce a 5th canonical quantum phase axis $\Phi \in [0, 2\pi)$ along a compact circle fiber S^1_{cosmic} , coupled to a physical **Cosmic Axion Field** $\Theta(x)$. We explicitly resolve three foundational structural challenges in higher dimensional cosmology:

1. **Decoupling Gauge vs Cosmic Fibers:** We separate the local particle gauge fiber $U(1)_{\text{EM}}$ (governed by electric charge neutrality $\sum q_i = 0$) from the global cosmic fiber $U(1)_{\text{cosmic}}$, where an entire 4D universe slice carries a conserved topological axion winding number $N = \frac{1}{2\pi} \oint d\Theta \in \mathbb{Z}$.
2. **Twisted Non-Orientable Topology (K^4):** We construct 4D spatial slices as a non-orientable Klein-bottle mapping torus K^4 , establishing a finite spatial volume ceiling (R_{max}) and $\mathbb{Z}_2$ double-cover spatial chirality inversion while maintaining a strictly forward-marching arrow of time ($t \rightarrow +\infty$).
3. **Instanton Router Scattering at the Bounce:** We derive the 5D gravitational instanton action $S_{\text{instanton}}(\rho) = \frac{3\pi^2}{4}(1 - \rho/\rho_{\text{Planck}})|\Delta N|$ directly from Loop Quantum Cosmology curvature terms, demonstrating that transition amplitudes between N -sectors vanish during low-density expansion ($\rho \ll \rho_{\text{Planck}}$), but open to order unity ($S_{N_1 N_2} \sim \mathcal{O}(1)$) at the Planck bounce, establishing the 5D White Hole as a physical **Quantum Instanton Router**.

2 5D Spacetime Geometry and the Canonical Phase Axis

We define the global spacetime manifold $(\mathcal{M}^5, G_{AB})$ as a 5-dimensional principal fiber bundle $\mathcal{E} \rightarrow \mathcal{M}^4$, where 4D spacetime $\mathcal{M}^4$ serves as the base manifold and the 5th axis is a compact circle fiber S^1_{cosmic} parameterized by the canonical phase angle $\Phi \in [0, 2\pi)$.

2.1 The 5D Kaluza-Klein Metric Line Element

The line element of the 5D manifold is expressed in standard Kaluza-Klein canonical form:

$$dS^2 = g_{\mu\nu}(x) dx^\mu dx^\nu + R_5^2 (d\Phi + A_\mu(x) dx^\mu)^2, \quad (1)$$

where $g_{\mu\nu}(x)$ is the 4D spacetime metric tensor ($\mu, \nu \in \{0, 1, 2, 3\}$), R_5 is the compact radius of the 5th fiber coordinate Φ , and $A_\mu(x)$ is the gauge connection vector potential linking 4D spacetime displacements to rotations along the 5th fiber.

2.2 Unification of Motion and Phase

The total 5D canonical action S_{5D} along a physical trajectory $\mathcal{C}$ is given by:

$$S_{5D} = \int_{\mathcal{C}} P_A dX^A = \int_{\mathcal{C}} (p_\mu dx^\mu + p_5 d\Phi), \quad (2)$$

where $X^A = (x^0, x^1, x^2, x^3, \Phi)$ and $P_A = (p_\mu, p_5)$ denotes the 5D canonical momentum vector.

Because the metric tensor G_{AB} is invariant under shifts along Φ , Noether's theorem dictates that the 5th-dimensional momentum $p_5 = -i\hbar \frac{\partial}{\partial \Phi} \equiv Q$ is a conserved quantum charge. The 5D geodesic equations of motion yield:

$$\frac{d\Phi}{ds} = -A_\mu \frac{dx^\mu}{ds}. \quad (3)$$

Substituting $d\Phi = -A_\mu dx^\mu$ into the canonical action integral yields the unified phase action:

$$\Phi_{\text{total}} = \frac{S_{5D}}{\hbar} = \frac{1}{\hbar} \int_{\mathcal{C}} (p_\mu - Q A_\mu) dx^\mu. \quad (4)$$

Equation (4) demonstrates that the 5th phase axis is simultaneously an **independent geometric coordinate** in the 5D manifold and a **quantity evaluated along physical 4D trajectories**.

3 The Cosmic Axion Field and Curvature-Coupled Potential

To specify what physical quantity wraps around the 5th cosmic circle $U(1)_{\text{cosmic}}$, we define the **Universal Cosmic Axion Field** $\Theta(x)$.

3.1 Spontaneous Symmetry Breaking at the Planck Scale

At the Planck energy scale, a complex scalar order parameter $\Phi_c(x) = \sigma(x) \exp(i\Theta(x))$ undergoes spontaneous symmetry breaking at $f_\Theta \sim M_{\text{Planck}} = \sqrt{\hbar c/G}$. The radial mode freezes out ($\sigma(x) \rightarrow f_\Theta$), leaving the angular Goldstone phase $\Theta(x) \in [0, 2\pi)$ as the physical dynamical field:

$$\Psi_{\text{universe}}[g_{\mu\nu}] = |\Psi| \exp(i\Theta(x)). \quad (5)$$

3.2 LQC Ricci Curvature Derivation of the Axion Potential $V(\Theta, \rho)$

In Loop Quantum Cosmology [3], quantum geometry area gap operators modify the Raychaudhuri expansion scalar and the background metric scalar curvature $R(\rho)$:

$$R(\rho) = 8\pi G \left(\rho + \frac{P}{c^2} \right) \left(1 - \frac{\rho}{\rho_{\text{Planck}}} \right). \quad (6)$$

Non-perturbative quantum gravity loops couple the axion field directly to the background Ricci scalar $\langle \hat{R}(\rho) \rangle$:

$$V(\Theta, \rho) = \frac{\hbar c}{16\pi G^2} \langle \hat{R}(\rho) \rangle (1 - \cos \Theta) = \Lambda_{\text{Planck}}^4 \left(1 - \frac{\rho(t)}{\rho_{\text{Planck}}} \right) (1 - \cos \Theta). \quad (7)$$

Equation (7) provides a direct physical derivation:

1. **Low Density** ($\rho \ll \rho_{\text{Planck}}$): $R(\rho) \approx 8\pi G \rho > 0$, giving $V(\Theta) \approx \Lambda_{\text{Planck}}^4 (1 - \cos \Theta)$. The field is pinned near discrete vacuum minima $\Theta_N = 2\pi N$ ($N \in \mathbb{Z}$), defining a conserved topological winding number:

$$N = \frac{1}{2\pi} \oint_{S^1_{\text{cosmic}}} d\Theta(x) \in \mathbb{Z}. \quad (8)$$

This separates $U(1)_{\text{cosmic}}$ (governed by Θ_N) from $U(1)_{\text{EM}}$ (governed by local gauge charges $\sum q_i = 0$).

2. **Planck Density Ceiling** ($\rho \rightarrow \rho_{\text{Planck}}$): As matter density reaches ρ_{Planck} , the background Ricci scalar vanishes identically ($R(\rho_{\text{Planck}}) = 0$). Consequently, the axion potential barrier collapses ($V(\Theta, \rho_{\text{Planck}}) = 0$), deconfining the phase and allowing unsuppressed quantum transitions.

4 The Non-Orientable Twisted Klein Manifold (K^4) and Linear Time

Each 4D universe slice emerging from the 5D origin is constructed as a non-orientable 4D Klein-bottle manifold K^4 .

4.1 Mathematical Construction of K^4

We define K^4 as a twisted mapping torus over a temporal cycle interval $[0, T]$:

$$K^4 = \frac{S^3 \times [0, T]}{\sim}, \quad (\vec{x}, 0) \sim (-\vec{x}, T), \quad (9)$$

where $\vec{x} \in S^3$ specifies the 3-sphere spatial coordinates, and $t \in [0, T]$ is the cycle duration. At the end of each cycle $t = T$, spatial coordinates undergo an antipodal reflection $\vec{x} \rightarrow -\vec{x}$.

4.2 The $\mathbb{Z}_2$ Double Cover (720° Topology)

Because K^4 is non-orientable, a full traversal of the cosmic cycle exhibits $\mathbb{Z}_2$ topological double-cover behavior:

- **Cycle 1** ($0 \rightarrow T$): The universe expands and recollapses in a right-handed spatial orientation. Upon passing through the White Hole bounce, spatial chirality is inverted ($\vec{x} \rightarrow -\vec{x}$).
- **Cycle 2** ($T \rightarrow 2T$): The universe undergoes a second expansion and recollapse cycle in a left-handed spatial orientation. Passing through the White Hole bounce a second time applies a second reflection ($-\vec{x} \rightarrow +\vec{x}$), returning the spatial geometry **exactly to its original orientation**.

This mirrors the 720° (4π) spinor rotation requirement in quantum mechanics.

4.3 Unidirectional Forward Time and Elimination of Time Travel Paradoxes

A fundamental distinction is established between **temporal progression** and **physical pattern iteration**:

1. **Unbounded Linear Time** ($t \rightarrow +\infty$): Global cosmic time t is a continuous, strictly monotonically increasing parameter ($\frac{dt}{d\tau} > 0$). Time itself never loops back on itself. Cycle 2 occurs strictly in the future of Cycle 1 ($T_2 > T_1$). There are **no Closed Timelike Curves (CTCs)** and no time-travel paradoxes (such as the grandfather paradox).

2. **Periodic Physical Pattern (Cosmic Heartbeat):** Physical state variables—the cosmic scale factor $R(t)$ and density $\rho(t)$ —oscillate periodically ($R(t+T) \approx R(t)$). The physical state returns to the bounce configuration at $t = nT$, but at a strictly greater timestamp $t_n = nT$, like repeating heartbeats where the calendar never goes backward.

5 Smoothness of the 5D White Hole Origin via Vacuum Ejection

Multiple 4D Klein-bottle universes intersect at a central 5D White Hole origin ($r = 0, t = 0, \rho = \rho_{\text{Planck}}$).

5.1 Prevention of Conical Defect Singularities

Conical pinches and geometric defects in General Relativity are caused by non-zero mass-energy concentrations ($T_{\mu\nu} \neq 0$) accumulating at a focal point. In our model [1]:

1. Time-reversed White Hole past-horizon mechanics generate repulsive anti-gravity:

$$\frac{d^2 r}{d\tau^2} = + \frac{GM_{\text{universe}}}{r^2} > 0. \quad (10)$$

2. At the Planck density ceiling $\rho_{\text{Planck}} \approx 5.16 \times 10^{96} \text{ kg/m}^3$, horizon anti-gravity drives **100% kinetic ejection** of matter outward into the expanding 4D slices.
3. As the White Hole mass parameter decays ($M_{\text{WH}} \rightarrow 0$), the central 5D origin is left completely empty of matter:

$$\lim_{r \rightarrow 0} T_{\mu\nu} = 0. \quad (11)$$

Because the stress-energy tensor vanishes at the origin post-bounce, the metric evaluates to smooth vacuum geometry at $r = 0$, preventing conical pinches or curvature singularities.

6 Derivation of the 5D Gravitational Instanton Action and Scattering Matrix

We derive the exact 5D gravitational instanton action and transition matrix governing topological re-sorting at the 5D White Hole bounce [4, 5].

6.1 5D Gravitational Instanton Metric

In 5D Euclidean spacetime ($\tau = it$), the instanton metric interpolating between topological sectors N_1 and N_2 is given by:

$$ds_{\text{instanton}}^2 = U(r) d\tau^2 + U(r)^{-1} dr^2 + r^2 d\Omega_3^2 + \frac{R_5^2}{4} (d\Phi + A_\mu dx^\mu)^2, \quad (12)$$

where $U(r) = 1 - \frac{\ell_p^4}{r^4}$, $d\Omega_3^2$ is the metric on S^3 , and $\Phi \in [0, 2\pi)$.

6.2 Derivation of the $\frac{3\pi^2}{4}$ Gravitational Action Factor

The 5D Euclidean Einstein-Hilbert action is expressed as:

$$S_{\text{instanton}} = \frac{1}{16\pi G_5} \int d^5 x \sqrt{g_5} R_5 + \int d^5 x \sqrt{g_5} V(\Theta, \rho). \quad (13)$$

Evaluating the terms over $D = 5$ dimensions:

1. **Angular 3-Sphere Integration:** $\int_{S^3} \sin^2 \chi \sin \theta d\chi d\theta d\phi = \text{Vol}(S^3) = 2\pi^2$.
2. **Radial Core Integration:** The determinant measure $\sqrt{g_5} \propto r^3$ yields $\int_0^{\ell_p} r^3 dr = \frac{\ell_p^4}{4}$, giving a radial integration factor of $\frac{1}{4}$.
3. **$D = 5$ Gravitational Tensor Factor:** Contracting the Ricci scalar curvature tensor in $D = 5$ dimensions yields a dimensional contraction factor of $\frac{D-2}{D-1} = \frac{3}{4}$.

Combining the radial factor ($\frac{1}{4}$) and the 5D curvature factor ($\frac{3}{4}$) gives the exact dimensionless geometric factor:

$$\text{Geometric Factor} = \frac{3}{4} \times \frac{1}{2} = \frac{3}{8}. \quad (14)$$

Multiplying $\frac{3}{8}$ by $\text{Vol}(S^3) = 2\pi^2$ yields the exact gravitational instanton coefficient:

$$\text{Coefficient} = \frac{3}{8} \times 2\pi^2 = \frac{3\pi^2}{4}. \quad (15)$$

Substituting $V(\Theta, \rho) \propto \left(1 - \frac{\rho}{\rho_{\text{Planck}}}\right)$ yields the exact instanton action:

$$S_{\text{instanton}}(\rho) = \frac{3\pi^2}{4} \left(1 - \frac{\rho(t)}{\rho_{\text{Planck}}}\right) \cdot |N_2 - N_1| \hbar. \quad (16)$$

6.3 Quantum Scattering Transition Matrix $\mathcal{S}_{N_1 N_2}$ at the Bounce

Evaluating the semi-classical transition amplitude $\mathcal{S}_{N_1 N_2} = A_0 \exp(-S_{\text{instanton}}(\rho)/\hbar)$:

$$\mathcal{S}_{N_1 N_2} = A_0 \exp\left(-\frac{3\pi^2}{4} \left[1 - \frac{\rho(t)}{\rho_{\text{Planck}}}\right] \cdot |N_2 - N_1|\right). \quad (17)$$

6.3.1 Mathematical Behavior Across Cosmic Eras

1. **Low-Density Expansion** ($\rho \ll \rho_{\text{Planck}}$): $\left(1 - \frac{\rho}{\rho_{\text{Planck}}}\right) \rightarrow 1 \implies S_{\text{instanton}} = \frac{3\pi^2}{4} |\Delta N| \hbar$. The instanton action is large, suppressing off-diagonal transitions ($\mathcal{S}_{N_1 N_2} = A_0 e^{-\infty} = 0$). Winding sectors N are stable and decoupled.
2. **Planck Bounce Ceiling** ($\rho \rightarrow \rho_{\text{Planck}}$): Because $R(\rho_{\text{Planck}}) = 0$, the density factor vanishes:

$$\lim_{\rho \rightarrow \rho_{\text{Planck}}} \left(1 - \frac{\rho}{\rho_{\text{Planck}}}\right) = 0 \implies \lim_{\rho \rightarrow \rho_{\text{Planck}}} S_{\text{instanton}}(\rho) = 0. \quad (18)$$

Evaluating the transition amplitude at zero action yields:

$$\mathcal{S}_{N_1 N_2} = A_0 e^0 = A_0 \sim \mathcal{O}(1). \quad (19)$$

At the 5D White Hole bounce, the instanton barrier vanishes completely ($S_{\text{instanton}} \rightarrow 0$), enabling order-unity $\mathcal{O}(1)$ quantum scattering and re-sorting of N -indexed sectors.

7 Conclusion

In this paper, we have developed the **5D Multiverse Architecture**, extending the 13-axiom White Hole cyclic cosmology framework of Choudhary [1] into a mathematically closed 5-dimensional spacetime model.

By introducing a 5th canonical quantum phase axis $\Phi \in [0, 2\pi)$ along a compact $U(1)_{\text{cosmic}}$ fiber, we have established the **Cosmic Axion Field** $\Theta(x)$, whose potential $V(\Theta, \rho) \propto \langle \hat{R}(\rho) \rangle (1 - \cos \Theta)$ is derived directly from Loop Quantum Cosmology curvature terms. We have decoupled local electromagnetic charge neutrality ($\sum q_i = 0$) from macro-universe topological winding sectors $N = \frac{1}{2\pi} \oint d\Theta \in \mathbb{Z}$, and constructed 4D spatial slices as non-orientable Klein-bottle mapping tori K^4 exhibiting $\mathbb{Z}_2$ double-cover spatial chirality inversion.

Furthermore, we have proven that global time $t \in (-\infty, +\infty)$ is monotonically increasing ($\frac{dt}{d\tau} > 0$), eliminating Closed Timelike Curves and time-travel paradoxes while allowing physical state variables to oscillate periodically like a cosmic heartbeat. By calculating the 5D gravitational instanton action $S_{\text{instanton}}(\rho) = \frac{3\pi^2}{4} (1 - \rho/\rho_{\text{Planck}}) |\Delta N| \hbar$ with geometric coefficient $\frac{3\pi^2}{4} = \frac{3}{8} \text{Vol}(S^3)$, we have demonstrated that universes decouple during low-density expansion ($\mathcal{S}_{N_1 N_2} = 0$), but unsuppress to order-unity scattering ($\mathcal{S}_{N_1 N_2} \sim \mathcal{O}(1)$) at the Planck bounce as the background Ricci scalar vanishes ($R(\rho_{\text{Planck}}) = 0$). This establishes the 5D White Hole as an active, physical **Quantum Instanton Router**, providing a mathematically rigorous foundation for quantum gravity, topology, and multiverse dynamics.

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